We have the equation

$$(P - p)\left[1 + \frac{r}{4}\right]^3 - p\left[1 + \frac{r}{4}\right]^2 - p\left[1 + \frac{r}{4}\right] - p = 0.$$

Putting

$$1 + \frac{r}{4} = x$$

and substituting value of P, we have the equation

$$294x^3 - 106x^2 - 106x - 106 = 0.$$

Solving by Horner's Method, we have

$$x = 1 + \frac{r}{4} = 1.04028,$$

$$r = .16112 = 16.112 \text{ per cent compounded quarterly.}$$

B (by arithmetic). A more elementary and more "practical" method is the method by trial and error. A few trials will show that the rate is something over 16 per cent.

First Trial. Taking the rate as 16 per cent and the annual premium as 100, we have the scheme,—

Annual premium due	100.00
First quarterly premium paid	26.50
	73.50
Interest for three months	2.94
	76.44
Second quarterly premium paid	26.50
	49.94
Interest for 3 months	2.00
	51.94
Third quarterly premium paid	26.50
	25.44
Interest for 3 months	1.02
	26.46
Fourth quarterly premium	26.50
First error	0.04

We see that 16 per cent is slightly too small.

Second trial. Taking the rate as 16.2 per cent., we have, in the same way as before, an error of + .03.

Forming a table

Rate	Error
16	−.04
16.2	+.03

By interpolation, the rate that will give zero error is

$$16 + \frac{4}{7} \times .2 = 16.114 \text{ per cent.}$$

If greater accuracy were required, repeat the computation with the last rate and interpolate again.

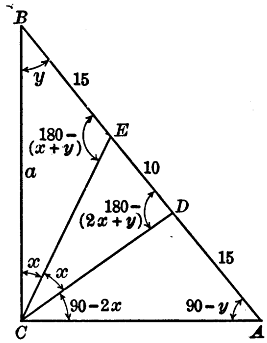
Also solved by G. N. ARMSTRONG, H. N. CARLETON, and H. L. OLSON.

2747 [1919, 72]. Proposed by DANIEL KRETH, Wellman, Iowa.

In the right triangle ABC, right angle C, we have given on the hypotenuse the segments $AD = 15$, $DE = 10$, $EB = 15$, and the angle DCE equal to the angle ECB. Find the angle DCE, and the sides AC and BC.

SOLUTION BY MARCIA LATHAM, New York City.

Let $BC = a$, $AC = b$, $DC = c$, $\angle DCE = \angle ECB = x$, $\angle ABC = y$. Then $\angle DCA = 90 - 2x$, $\angle EDC = 180 - (2x + y)$, and $\angle BEC = 180 - (x + y)$.



$$\sin y = b/40 \qquad \text{and} \qquad \cos y = a/40. \tag{1}$$

Since EC bisects $\angle BCD$,

$$a/c = 15/10 = 3/2. \tag{2}$$

In the triangle, DCA, by the law of sines,

$$c/15 = \cos y/\cos 2x. \tag{3}$$

Combining (1), (2), and (3), $2a/45 = a/40 \cos 2x$, or $\cos 2x = 9/16$, $x = 1/2 \cos^{-1} 9/16 = 27° 53' 8''$. In the triangle, BCD, by law of sines, $\sin(2x + y)/\sin y = a/c$. Whence, by (1) and (2)

$$\sin(2x + y) = 3b/80. \tag{4}$$

Also, $a/25 = \sin(2x + y)/\sin 2x$; whence, by (4), $\sin 2x = 15b/16a$. But $\sin^2 2x + \cos^2 2x = 1$. Then $(15b/16a)^2 + (9/16)^2 = 1$; whence

$$b^2 = 7a^2/9. \tag{5}$$

Now, in the triangle, ABC, $a^2 + b^2 = (40)^2$; whence, by (5), $a^2 + 7a^2/9 = 1{,}600$; whence, $a = 30$ and from (5), $b = 10\sqrt{7}$.

Also solved by A. M. HARDING, POLYCARP HANSEN, C. E. HORNE, R. A. JOHNSON, ELMER LATSHAW, E. W. MARTIN, LOUIS ORDANKSY, A. PELLETIER, J. L. RILEY, H. M. ROESER, L. SMITH, D. L. STAMY, H. TSAI, and L. G. WELD.

2760 [1919, 124]. Proposed by CHARLES N. SCHMALL, New York City.

In an arithmetical progression, if s_n be the sum of the first n terms, s_{2n} the sum of the first $2n$ terms, and s_{3n} the sum of the first $3n$ terms of the same series, prove that $s_{2n} - s_n = \frac{1}{3}s_{3n}$.

SOLUTION BY EMMA M. GIBSON, Springfield (Mo.) High School.

The sum of n terms of an arithmetical progression is expressed by the formula

$$s_n = \frac{n(a_1 + a_n)}{2},$$

where a_1 and a_n are the first and nth terms, respectively.

Hence, $s_n = n(a_1 + a_n)/2$, $s_{2n} = 2n(a_1 + a_{2n})/2$, and $s_{3n} = 3n(a_1 + a_{3n})/2$ are the sums of the first n terms, the first $2n$ terms, and the first $3n$ terms, respectively. Now $a_{2n} = a_n + nd$, $a_{3n} = a_{2n} + nd = a_n + 2nd$, d being the common difference.

Then $s_{2n} = 2n(a_1 + a_n + nd)/2$ and $s_{3n} = 3n(a_1 + a_n + 2nd)/2$ and

$$s_{2n} - s_n = 2n(a_1 + a_n + nd)/2 - n(a_1 + a_n)/2 = n(a_1 + a_n + 2nd)/2 = s_{3n}/3.$$

Also solved by R. D. BOHANNAN, H. L. BRIDGES, JR., H. N. CARLETON, W. F. CHENEY, JR., P. J. DA CUNHA, H. C. GOSSARD, WILLIAM HERBERG, C. N. MILLS, LOUIS O'SHAUGHNESSEY, H. L. OLSON, A. PELLETIER, J. B. REYNOLDS, I. S. SUN, and ELIJAH SWIFT.

2768 [1919, 171]. Proposed by PAUL CAPRON, U. S. Naval Academy.

Given the center, a focus, and a point of a conic, construct geometrically the circle of curvature at the point.